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植被科学与多元向量分析

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摘要：提出植被分析的新数学方法——多元向量分析法。植被群落是由多种植物组成的。植被状态可以用多维物种空间的点，或连接空间点和原点的多元向量来表示。向量同时具有量值和方向。向量的量值(长度)表示植被所含物质，能量，信息的总量，而方向表示这个总量在各物种间的分配。在射影空间里，同一射线上的点表示成分相同的植被(或代表相同植被的点组成射线)。用余弦表示的向量方向是植被数量化，进而施行植被成分分析的关键。向量分析既可以用来进行植被分类，也可以用于植被动态监测。

关键词：超球面模型；多维变量空间；多元向量；余弦向量；向量分析；植被分析

Vegetation Science and Vector Analysis

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Abstract: Vegetation is a community of plants. Vegetation with multi-plant species can be described by points in multi-species space, or multi-component vectors, *m*-vectors. An *m*-vector is a quantity that has magnitude and direction. The magnitude of the vector expresses the mass of the vegetation, while the direction of the vector expresses the composition of the vegetation. The directions of *m*-vectors are essential to quantify plant communities. Vector directions can be used for both vegetation classification and vegetation dynamic analysis.

Key words: multi-dimensional sphere model; *m*-space; *m*-vector; *m*-cosine; vegetation composition; community

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1 Introduction

As part of the discussion on Ecolog-L about "A Critique for Ecology", a commentary entitled "A Critique of Matrix Solutions for Ecology" was published in Bulletin of the Ecological Society of America, April, 1998[1]. Since then, the author has received several responses about the commentary. Some of these responses suggested that a vector is a kind of matrix and that the author seemed to be inflating the importance of the vector. This essay is to point out the distinction between the usage of vectors and matrices in vegetation analysis. Although both $m \times 1$ matrices and multi-component vectors, *m*-vectors, are multicomponent arrays and an *m*-vector can be treated as $m \times 1$ matrix sometimes, the *m*-vector is not an $m \times 1$ matrix. In fact, an *m*-vector is more than an $m \times 1$ matrix. Vectors not only have the magnitude, the same as a matrix, but also have a direction, and are thus different from a matrix. By definition, a vector is a quantity that has magnitude and direction[2]. In Chinese, a vector is called *xiangliang*, which

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means direction (i. e. , *xiang*), and magnitude (i. e. , *liang*). It is the direction that makes vectors different from other quantities. It also makes a difference in applying an m-array to vegetation analysis, with or without direction[3,4].

2 An Example for Vegetation

The following is an example of what the direction of an m-vector means to vegetation science. In vegetation science, scientists consider the composition of the vegetation to mean more than the total biomass[3,4]. Imagine, a three dimensional space (3-space), where the three dimensions are trees, shrubs, and grasses. In this 3-space, vector $\mathbf{A}=(3, 1, 0)$ is different from vector $\mathbf{B}=(0, 1, 3)$, although the two have the same magnitude equal to the square root of 10. How does one distinguish two vectors when their magnitudes are the same? The answer is to use the position of the points in 3-space, or the direction of the position-vectors in the 3-space. The position of point A, or direction of the vector OA (where O is the origin, and A is the point in 3-space, OA is the position vector. We use **A** in bold to express the vector OA in this essay) is closer to the first axis of Trees. Consequently, **A** can be classified as a woodland. On the other hand, **B** is closer to the third axis of Grasses, and may be classified as grassland. The two 3-vectors, $\mathbf{A}=(3, 1, 0)$ and $\mathbf{B}=(0, 1, 3)$, represent a different composition and are different types of vegetation. For the same reason, vector $\mathbf{2A}=(6, 2, 0)$ in the same 3-space may be classified as woodland the same as $\mathbf{A}=(3, 1, 0)$, since **A** and **2A** have the same composition ratio, yet **A** and **2A** have a difference in magnitude. This difference, however, is not essential to separate the two in vegetation science. This difference may be caused by different sampling areas or different sampling times. For example, **2A** might have twice the sampling area as **A** did, or **2A** might have a wetter sampling season than **A** did. However, those are minor factors, and would not change the vegetation classification. What is essential to vegetation analysis is whether **A** and **2A** have the same composition; that is, whether **A** and **2A** have the same direction in multi-species space (m-space), in other words, whether they are colinears[3, 4].

Generally speaking, all the points representing the same vegetation are located on the same ray in the m-space. In other words, all m-vectors representing the same vegetation have the same direction in the m-space, or are colinears. Just as direction is essential to a vector, composition is essential to vegetation analysis. Different directions in m-space equate to different vegetation, and the same direction means the same vegetation. Furthermore, any compositional change in vegetation can be expressed by changes in vector direction. Therefore, vegetation dynamics, changes in composition, can be, and can only be, expressed by vector rotation in the m-space. A vegetation scientist can monitor vegetation change by monitoring the rotation of the state vector of the vegetation in the m-space . This may be the key issue for a vegetation inventory and monitoring program. The direction of an m-vector in the m-space can be expressed by the vector's m-cosine values[3, 5~7]:

$$\text{Direction } \mathbf{A} = \text{cosine} < \mathbf{A} > = \text{cosine } \mathbf{A}_{(i)} = \mathbf{A}_{(i)} / |\mathbf{A}_{(i)}| \quad i = 1, 2, \cdots, m$$

where $|\mathbf{A}_{(i)}|$ is the magnitude of the vector, the square root of the sum of the squares (in another field, it is also called Shang Gao Index), and cosine$<\mathbf{A}>$ means cosine m-angle **A**, or direction of vector **A** in m-space.

For example, the direction of vector **OA** and **O2A** above can be expressed as

$$\text{Direction } \mathbf{A} = \text{cosine } \mathbf{A}_{(i)} = \mathbf{A}_{(i)} / |\mathbf{A}_{(i)}|$$

$$= \text{Direction } \mathbf{2A} = \text{cosine } \mathbf{2A}_{(i)} = \mathbf{2A}_{(i)} / |\mathbf{2A}_{(i)}|$$
$$= (3/\sqrt{(10)},\ 1/\sqrt{(10)},\ 0/\sqrt{(10)})$$
$$= (6/\sqrt{(40)},\ 2/\sqrt{(40)},\ 0/\sqrt{(40)}) = \mathbf{A}'$$

We notice that the cosine $\langle \mathbf{A} \rangle$ =cosine $\langle \mathbf{2A} \rangle$ =cosine $\langle \mathbf{A}' \rangle$ is also a 3-vector, when **A**, **2A**, and **A′** are 3-vectors. These are their projections on the three axes. Furthermore, the relation between any two vectors is expressed by a scalar which is the cosine value between the two vectors, or their projection to each other[8].

For example, the

$$\text{cosine} \langle \mathbf{OA}, \mathbf{O2A} \rangle = \text{cosine} \angle A - O - 2A = 1, \text{ but the}$$
$$\text{cosine} \langle \mathbf{OA}, \mathbf{OB} \rangle = \text{cosine} \angle \mathrm{A} - \mathrm{O} - \mathrm{B} = 1/10 = 0.1$$

Using m-vector and m-space, vegetation scientists can build a standard vegetation classification system[7]. This system can accept as many species as required, and can handle as many samples as are available. The differences in sampling size, such as $10\mathrm{m}^2$ versus $100\mathrm{m}^2$, sampling shape, such as line versus square, and measurement, such as, weight versus cover, would be filtered out by standardization. Thus, the only remaining information is the composition.

As different vectors can have the same magnitude, magnitude by itself tells us little about the composition of a vegetation. On the other hand, after knowing direction, a vegetation scientist can determine any individual component or/and the vector magnitude given only a single component. For example, if we know the type of vegetation was woodland, and the components of the vegetation were 3, 1, and 0, or direction of the state vector in the 3-space is $(3/\sqrt{10}, 1/\sqrt{10}, 0/\sqrt{10})$, as discussed above, then given a shrub value of 2, we can project the vegetation as (6, 2, 0), and the magnitude of the vegetation would be calculated as the square root of 40. This is the principle that the Multi-Dimensional Sphere Model (MDSM) uses for system dynamic analysis. This theory and the model have been tested with artificial data[3], vegetation data[9,10], as well as stock market data[11,12] in both China and the United States of America.

3 Conclusion

Vegetation occurs as multi-component communities, and communities can only be accurately expressed as multi-component vectors, m-vectors. An m-vector is fully described by its magnitude and direction. Vegetation with m species is fully described by the magnitude and direction of its m-state vector. The magnitude of the vector expresses the amount of material, energy, or information that the vegetation contains, while the direction expresses the distribution of those material, energy, or information among the m components. The direction of a vector can be expressed by cosine values. The cosine value between two vectors expressing their relation is a scalar. However, the cosine values associated with each of the m axes expressing the vector's direction in the m-space is itself an m-vector. More specifically, it is the projection of the vegetation on the unit multidimensional sphere.

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